



PROGRESSION IN CALCULATION

CHARLTON KINGS INFANTS' SCHOOL



Charlton Kings
Infants' School



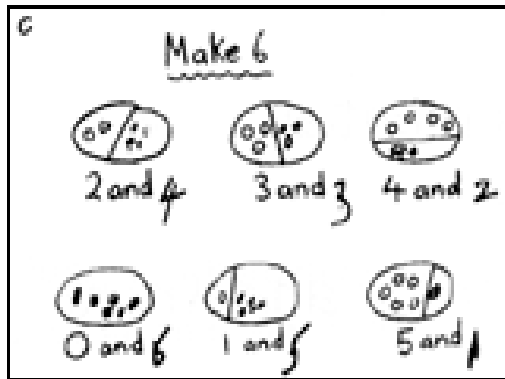
ADDITION



NB. CHILDREN MAY PROGRESS THROUGH THE STAGES AT DIFFERENT RATES.

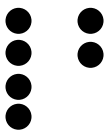
Stage 1

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures, etc.



Children use jottings to complete simple addition sentences. They draw dots under each number and count them to find the total.

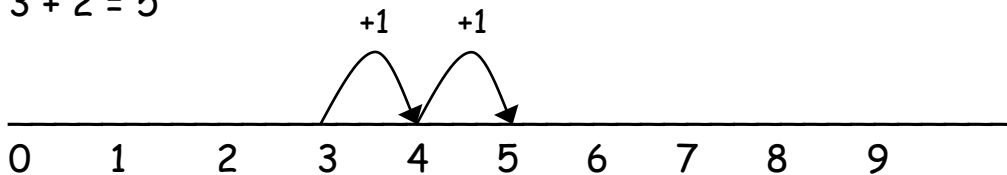
$$4 + 2 = 6$$



Stage 2

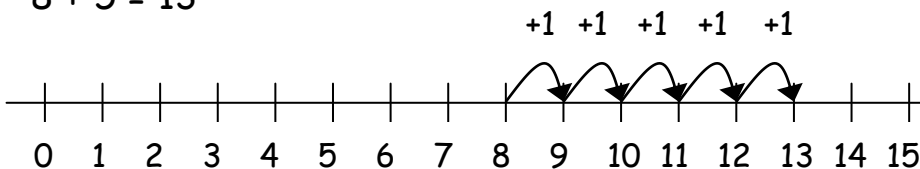
They use number lines and practical resources to support calculation and teachers demonstrate the use of the number line.

$$3 + 2 = 5$$



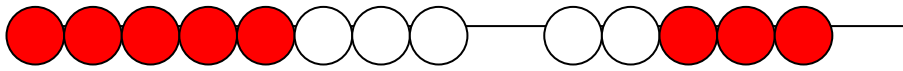
Children then begin to use numbered lines to support their own calculations using a numbered line to count on in ones.

$$8 + 5 = 13$$





Bead strings or bead bars can be used to illustrate addition including bridging through ten by counting on 2 then counting on 3.

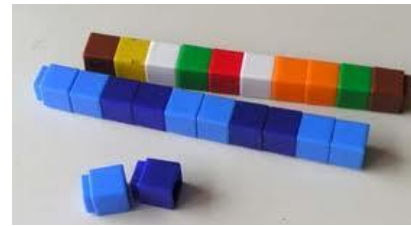


Children use jottings to complete more complex addition sentences. They use sticks/rods to represent the tens and dots to represent the ones in a two digit number. This is also demonstrated to the children physically using 'unifix' cubes.

$$13 + 22 = 35$$



The number 22 represented with 'unifix' cubes to demonstrate tens and ones



A hundred square can be used to demonstrate addition by counting on. Initially children will use the hundred square to count on in ones. When they are more familiar with the hundred square they employ faster strategies (demonstrated below). The hundred square is used in every year group at Charlton Kings Infants' School.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

$$37 + 12 = 49$$

$$37 \text{ add ten} = 47$$

$$47 \text{ add two} = 49$$

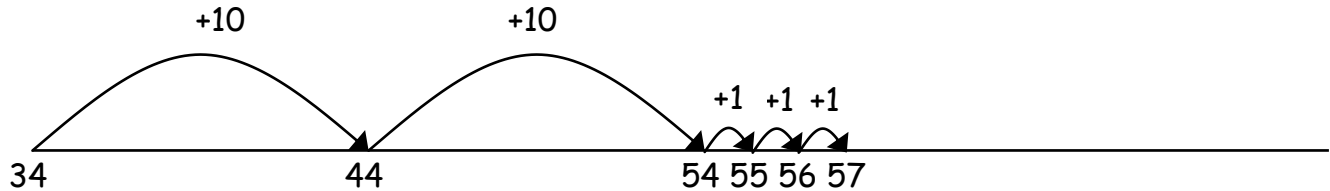


Stage 3

Children will begin to use 'empty number lines' themselves starting with the larger number and counting on.

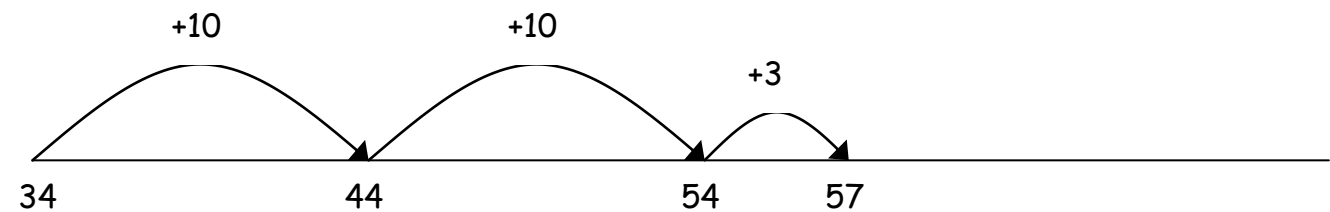
- ✓ First counting on in tens and ones.

$$34 + 23 = 57$$



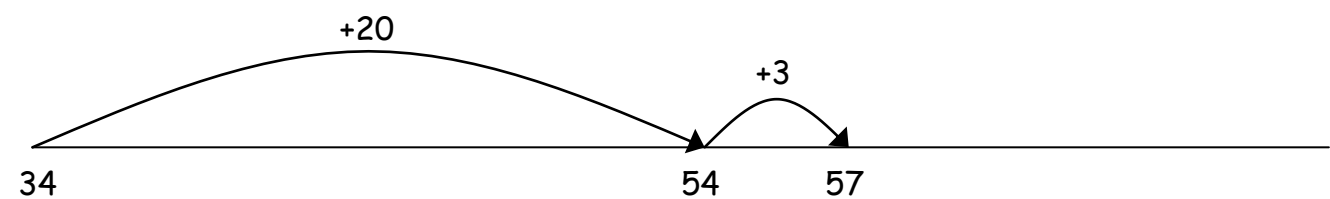
- ✓ Then helping children to become more efficient by adding the ones in one jump.

$$34 + 23 = 57$$



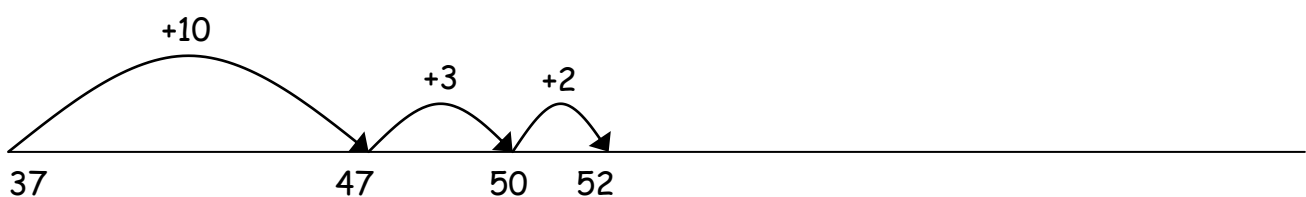
- ✓ Followed by adding the tens in one jump and the ones in one jump (by using the known fact $4 + 3 = 7$).

$$34 + 23 = 57$$



- ✓ Bridging through ten can help children become more efficient (by using the known fact $3 + 2 = 5$).

$$37 + 15 = 52$$



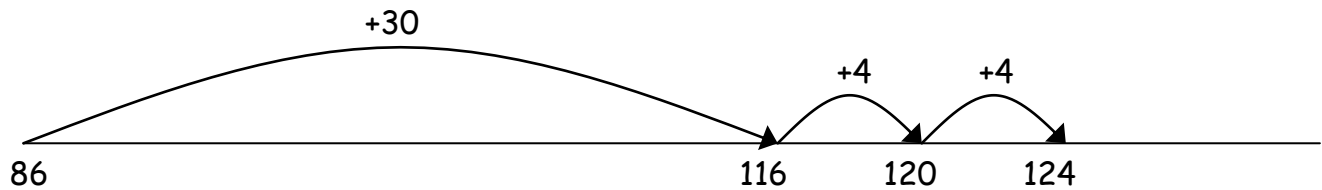


Stage 4

Children will continue to use empty number lines with increasingly large numbers, including compensation where appropriate.

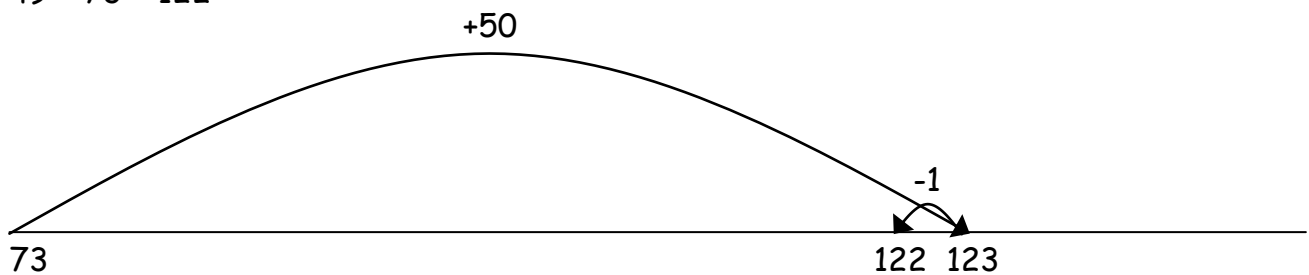
- ✓ Count on from the largest number irrespective of the order of the calculation.

$$38 + 86 = 124$$



- ✓ Compensation

$$49 + 73 = 122$$



Stage 5

- ✓ Partitioning and Recombining

Partition the number into tens and ones. E.g. $42 \rightarrow 40 + 2$

$$42 + 29 = \begin{array}{r} 40 + 2 \\ 20 + 9 \end{array}$$

Add the tens $40 + 20 = 60$

Add the ones $2 + 9 = 11$

$$60 + 11 = 71$$



SUBTRACTION



NB. CHILDREN MAY PROGRESS THROUGH THE STAGES AT DIFFERENT RATES.

Stage 1

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures etc.



Children use jottings to complete simple subtraction sentences. They draw dots under the first number and then cross out the relevant amount. Finally they count the remaining dots.

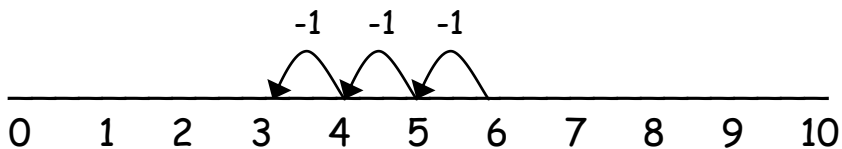
$$5 - 2 = 3$$



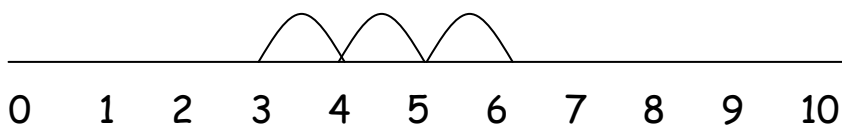
Stage 2

They use number lines and practical resources to support calculation. Teachers demonstrate the use of the number line.

$$6 - 3 = 3$$



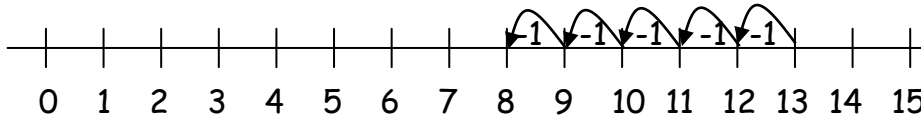
The number line should also be used to show that $6 - 3$ means the 'difference between 6 and 3' or 'the difference between 3 and 6' and how many jumps they are apart.





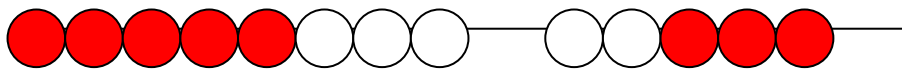
Children then begin to use numbered lines to support their own calculations - using a numbered line to count back in ones.

$$13 - 5 = 8$$



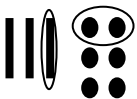
Bead strings or bead bars can be used to illustrate subtraction including bridging through ten by counting back 3 then counting back 2.

$$13 - 5 = 8$$



Children use jottings to complete more complex subtraction sentences. They use sticks/rods to represent the tens and dots to represent the ones in a two digit number. They then circle or cross out the number they are subtracting (demonstrated below). This is sometimes demonstrated to the children physically using 'unifix' cubes.

$$36 - 12 = 24$$



A hundred square can be used to demonstrate subtraction by counting back. Initially children will use the hundred square to count back in ones. When they are more familiar with the hundred square they employ faster strategies (demonstrated below). The hundred square is used in every year group at Charlton Kings Infants' School.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

$$58 - 23 = 35$$

58 subtract twenty = 38
38 subtract three = 35



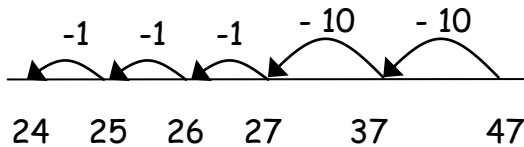
Stage 3

Children will begin to use empty number lines to support calculations.

Counting back

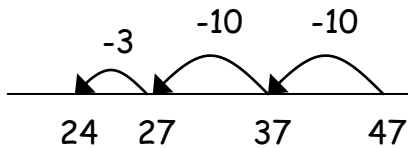
- ✓ First counting back in tens and ones.

$$47 - 23 = 24$$



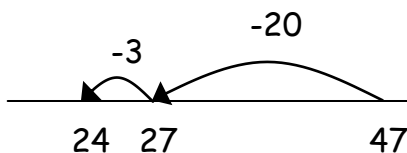
- ✓ Then helping children to become more efficient by subtracting the ones in one jump (by using the known fact $7 - 3 = 4$).

$$47 - 23 = 24$$



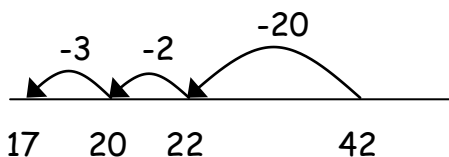
- ✓ Subtracting the tens in one jump and the ones in one jump.

$$47 - 23 = 24$$



- ✓ Bridging through ten can help children become more efficient.

$$42 - 25 = 17$$



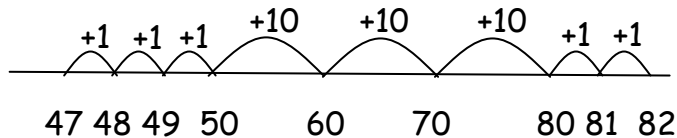


Counting on

If the numbers involved in the calculation are close together or near to multiples of 10, 100 etc, it can be more efficient to count on.

Count up from 47 to 82 in jumps of 10 and jumps of 1. Total the amount that was counted on e.g 35.

$$82 - 47 = 35$$



Stage 4

✓ Partitioning and Recombining

Partition the number into tens and ones. E.g. $89 \rightarrow 80 + 9$

Subtract the tens from the tens and then the ones from the ones.

$$89 - 57 = \begin{array}{r} 80 + 9 \\ 50 + 7 \end{array}$$

$$\text{Subtract the tens} \quad 80 - 50 = 30$$

$$\text{Subtract the ones} \quad 9 - 7 = 2$$

$$30 + 2 = 32$$



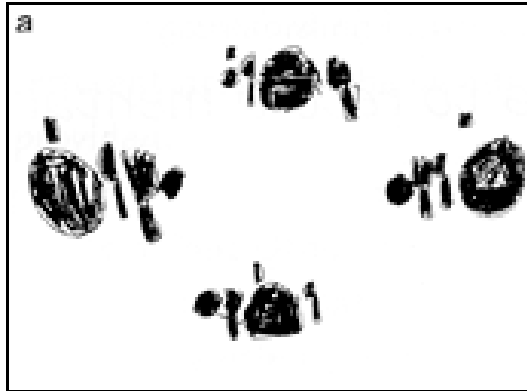
MULTIPLICATION



NB. CHILDREN MAY PROGRESS THROUGH THE STAGES AT DIFFERENT RATES.

Stage 1

Children will experience equal groups of objects and will count in 2s and 10s and begin to count in 5s. They will work on practical problem solving activities involving equal sets or groups.



Stage 2

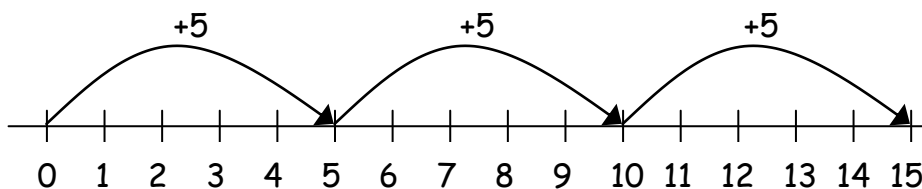
Children will develop their understanding of multiplication and use jottings to support calculation:

✓ Repeated addition

3 times 5 is $5 + 5 + 5 = 15$ or 3 lots of 5 or 5×3

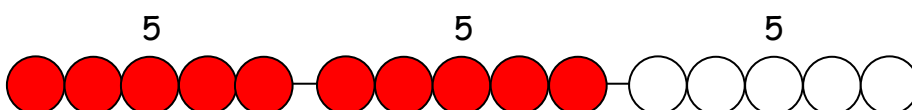
Repeated addition can be shown easily on a number line:

$$5 \times 3 = 5 + 5 + 5$$



and on a bead bar:

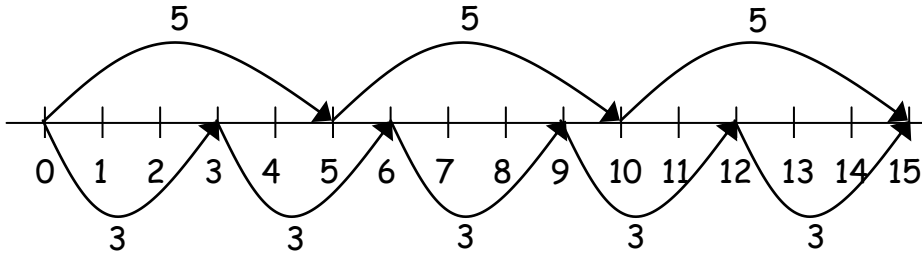
$$5 \times 3 = 5 + 5 + 5$$





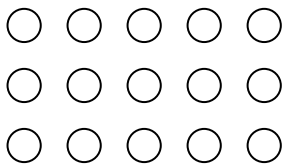
✓ Commutativity

Children should know that 3×5 has the same answer as 5×3 . This can also be shown on the number line.



✓ Arrays

Children should be able to model a multiplication calculation using an array. This knowledge will support with the development of the grid method.



$$5 \times 3 = 15$$

$$3 \times 5 = 15$$

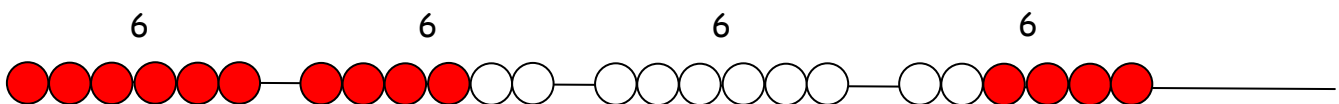
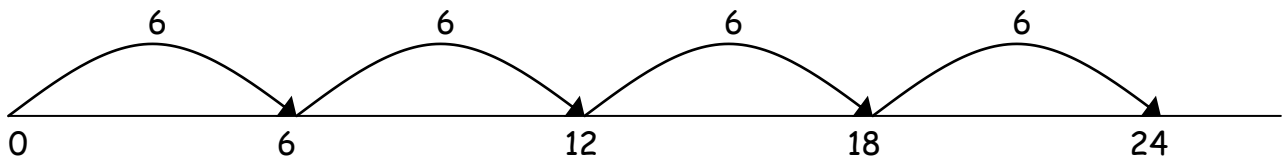
Stage 3

Children will continue to use:

✓ Repeated addition

4 times 6 is $6 + 6 + 6 + 6 = 24$ or 4 lots of 6 or $6 \times 4 = 24$

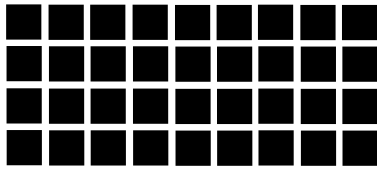
Children should use number lines or bead bars to support their understanding.





✓ Arrays

Children should be able to model a multiplication calculation using an array. This knowledge will support with the development of the grid method.



$$9 \times 4 = 36$$

$$9 \times 4 = 36$$

✓ Grid method

Some children will move on from an array to using the grid method.

X	10	2
5	50	10

$$\begin{aligned}
 12 \times 5 &= \\
 10 \times 5 &= 50 \\
 2 \times 5 &= 10
 \end{aligned}$$

$$50 + 10 = 60$$

The grid method can be used for more complex multiplications.

X	30	5
10	300	50
2	60	10

$$\begin{aligned}
 35 \times 12 &= \\
 30 \times 10 &= 300 \\
 5 \times 10 &= 50 \\
 30 \times 2 &= 60 \\
 5 \times 2 &= 10
 \end{aligned}$$

$$300 + 50 = 350$$

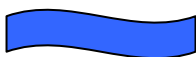
$$60 + 10 = 70$$

$$350 + 70 = 420$$

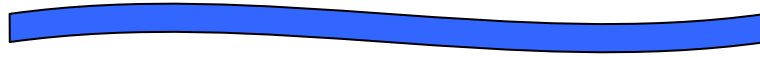
Children will also develop an understanding of:

✓ Scaling

e.g. Find a ribbon that is 4 times as long as the blue ribbon



5 cm



20 cm



- ✓ **Using symbols to stand for unknown numbers to complete equations using inverse operations**

$$\square \times 5 = 20$$

$$3 \times \triangle = 18$$

$$\square \times 8 = 32$$



DIVISION



NB. CHILDREN MAY PROGRESS THROUGH THE STAGES AT DIFFERENT RATES.

Stage 1

Children will understand equal groups and share items out in play and problem solving. They will count in 2s and 10s and later in 5s.

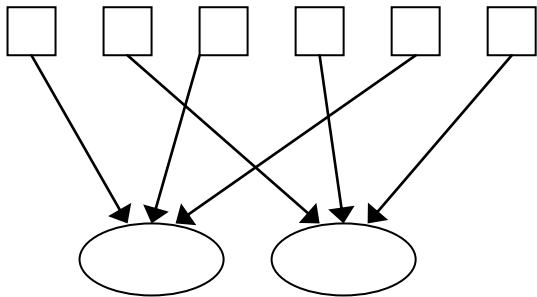


Stage 2

Children will develop their understanding of division and use jottings to support calculation.

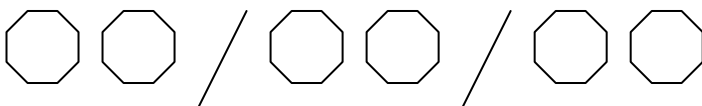
✓ **Sharing equally**

6 sweets shared between 2 people, how many do they each get?



✓ **Grouping or repeated subtraction**

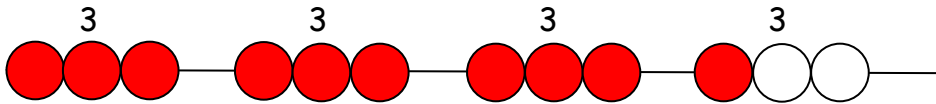
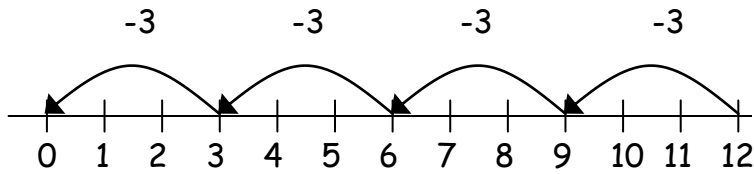
There are 6 sweets, how many people can have 2 sweets each?





✓ **Repeated subtraction using a number line or bead bar**

$$12 \div 3 = 4$$



The bead bar will help children with interpreting division calculations such as $10 \div 5$ as 'how many 5s make 10?'

✓ **Using symbols to stand for unknown numbers to complete equations using inverse operations**

$$\square \div 2 = 4$$

$$20 \div \triangle = 4$$

$$\square \div \triangle = 4$$

Stage 3

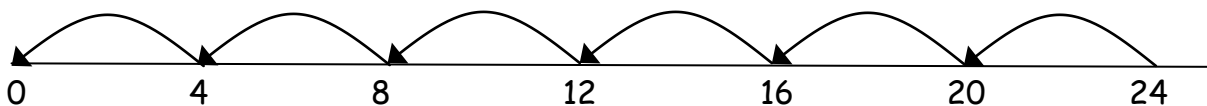
Emphasis in 'Stage 3' is on grouping rather than sharing.

Children will continue to use:

✓ **Repeated subtraction using a number line**

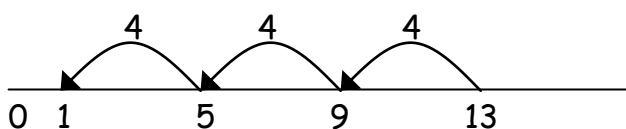
Children will use an empty number line to support their calculation by counting how many jumps backwards they have made.

$$24 \div 4 = 6$$



Children should also move onto calculations involving remainders.

$$13 \div 4 = 3 \text{ r } 1$$





✓ Using symbols to stand for unknown numbers to complete equations using inverse operations

$$26 \div 2 = \square$$

$$24 \div \triangle = 12$$

$$\square \div 10 = 8$$